Question Bank

BA. B.Ed. I

1. . Find the lines joining the origin to the points of intersection of $3x^2+4xy-4x+1=0$ and $2x+y-1=0$, and show that they are perpendicular.
2. Find the centre and radius of the circle $x^2+y^2-2x+4y=8$.
3. Find the degree and the leading coefficient of $(x^2-5x+10)(x^3-5x+9)=0$.
4. Let A=[a_{ij}] be a square matrix over **C**. Then A is skew-Hermitian if $A^\theta = A$.
5. Prove that every skew-symmetric matrix of odd order has rank less than its order.
6. Use Horner's method of synthetic division to divide

   $x^3 - 4x^2 + x + 6$ by $2x+3$
7. Use method of synthetic division to express

   $f(x) = x^4 - 3x^2 + 7x + 1$ as a polynomial in powers of $(x-3)$.
8. Without actual division, find the remainder when
   a. $x^4 - 3x^3 - 4x^2 - 6x + 15$ is divided by $(x-1)$.
   b. Find the quotient and remainder when f(x) is divided by g(x)

      $f(x) = x^4 + x^3 - x^2 + 1$, $g(x) = x^2 + 1$
9. Show that there exist no complex number a,b such that

   $(5-9a)+(3a+2b)x+(a-4b)x^2=2x-5x^2$
10. For what values of a and b is the polynomial

    $f(x) = x^2+(a^2+b)x+b-9$ the negative of the polynomial

    $g(x) = -x^2-5ax+b-3$.

11. Differentiate the following w.r.t.x:

    (a).log $(\tanh^{-1}\frac{x}{2})$ (b). $x^{\sinh x}$
12. Evaluate $[tan\frac{\pi}{4} + x\}^{\frac{1}{x}}$
13. Find nth derivative of $\tan^{-1}\frac{x}{a}$
14. If $y=\frac{logx}{x}$ prove that $y_n = \frac{(-1)^n n!}{x^{n+1}}[\log x - 1 - \frac{1}{2} - \frac{1}{3} - \ldots - \frac{1}{n}]$
15. Show that the matrix $A=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is diagonalization over C also find an invertible matrix P over C such that $P^{-1}AP$ is diagonal matrix.

16. For what value of λ, does the following system of equations

have a solution: x+y+z=1, x+2y+4z=λ, x+4y+10z=λ^2. Find also the solution in each case.

17. Verify Cayley-Hamilton theorem for the matrix A= $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{bmatrix}$ hence find A^{-1} .

18. Solve completely the system of equation:
X-2Y-Z=0, 2X+5Y+2Z=0, X+4Y+7Z =0, X+3Y+3Z=0

19. Express the matrix A = $\begin{bmatrix} 1+i & 2i & 3 \\ 0 & 2-3i & 3-4i \\ 5 & -7i & 0 \end{bmatrix}$

as the sum of a Hermitian and a skew-Hermitian matrix.

20. Through what angle should the axis be rotated so that the mixed term may disappear from the equation: $17X^2 - 16XY + 17Y^2$-225=0? Also find the transformed equation.

21. .Find the angle between the pair of straight lines represented by
$X^2 + XY - 6Y^2 + 7X + 31Y - 18 = 0$. Also find the equation of the pair of straight lines parallel to these and passing through the point (1,2) .

22.)Two circle each of radius 5 units touch each other at the point (1,2) of the equation of their common tangent is 4x+3y=10 find the equation of the circle.

23. .show that the point (x,y)given by x=$\frac{2at}{1+t^2}, y = \frac{a(1-t^2)}{(1+t^2)}$ (a is a given real number)lies on a circle for all real value of t.

B.A./B.Sc-II
Paper-1(Advanced Calculus)

Q1. If $f(x,y) = (x + y, (x + y)^2)$, evaluate $J_f(x,y)$.

Q2. Let $f(x,y) = (sinx, cosy)$ and $g(x,y) = (x^2, y^2)$.Evaluate J_F(x,y),where F=fog and verify the result by direct calculation.

Q3. State and prove Euler's theorem on homogeneous function of two variables.

Q4. Show that the volume of the largest parallelopiped that can be inscribed in the sphere $x^2 + y^2 + z^2 = a^2$

is $\frac{8a^2}{3\sqrt{3}}$.

Q5. Use Taylor's theorem to expand $x^2y - 3y + 3$ in power of $(x + 1)$ and $(y - 2)$.

Q6. Show that $f(x,y) = cos(x + y)$ is differentiable at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

Q7. If V=r^m, where $r = \sqrt{x^2 + y^2 + z^2}$, Show that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m + 1)r^{m-2}$.

Q8. Examine for the maximum and the minimum value of function $f(x,y) = sinx + siny + sin(x + y)$.

Q9. If $u = sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, then using Euler's theorem prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3\tan u = 0$.

Q10. Find the extreme values of the function $f(x,y) = (x - y)^4 + (y - 1)^4$.

Q11. Find all the points of maxima and minima of the function $f(x,y) = x^3 + y^3 - 63(x + y) + 12xy$. Also discuss the saddle points(If any) of the function.

Q12. If V= $f(r)$, where $r = \sqrt{x^2 + y^2}$, Prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.

Q13. Show that the functions $u = x + y + z, v = yz + zx + xy, w = x^3+y^3+z^3 - 3xyz$ are not independent of one another. Also find the relation between them.

Q14. Expand by Taylor's theorem, $x^4 + x^2y^2 + y^4$ about the point (1, 1) upto the terms of second degree.

Q15. If z=$\tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial z}{\partial x}+ y\frac{\partial z}{\partial x}=\frac{1}{4}\sin 2z$.

Q16. Let F: $R^3 \rightarrow 1R$ be defined by $F(x,y,z) = xyz$. Determine x, y, z for maximum of F subject to condition $xy + 2yz + 2zx = 108$.

Q17. If $u_1 = x_1 + x_2 + x_3 + x_4, u_1 u_2 = x_2 + x_3 + x_4$, $u_1 u_2 u_3 = x_3 + x_4$ and $u_1 u_2 u_3 u_4 = x_4$, Show that $\frac{\partial(x_1, x_2, x_3, x_4)}{\partial(u_1, u_2, u_3, u_4)} = u_1{}^3\, u_2{}^2\, u_3$.

Q18. If $H = f(y-z, z-x, x-y)$, prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$.

Q19. Find the extreme value(if any) of the function $f(x,y) = y^2 + x^2 y + ax^4; a \neq \frac{1}{4}$.

Q20. If $u^3 + v + w = x + y^2 + z^2$

$u + v^3 + w = x^2 + y + z^2$

$u + v + w^3 = x^2 + y^2 + z$

Prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1-4(yz+zx+xy)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}$

Q21. Let $f: R^3 \rightarrow R^2$ be defined by
$f(x,y,z) = (x, e^z \sin(x+y))$ and $g: R^2 \rightarrow R$ be defined by
$g(\xi, \eta) = \xi^2 + \eta^2$. Evaluate $F_x(x,y,z), F_y(x,y,z)$ and $F_z(x,y,z)$ by chain rule and verify by direct calculations, where $F = gof$.

Q22. Using Lagrange's method of multipliers, find the maximum and minimum values of x+y+z subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1, z = x + y$.

Q23. Find the minimum value of the function $u = x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$

Q24. If z=$f(u,v)$, where $u = e^x \cos y, v = e^x \sin y$, Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$

Q25. Use Taylor's theorem to expand $x^2 y + 3y - 2$ in powers of $x - 1$ and $y + 2$.

Q26. Find the extreme values of $x^2 + y^2 + z^2$ subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$

Q27. If $u = e^{xyz}$, Show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$.

Q28. State and prove Taylor's Theorem for Functions of Two Variables.

Q29. Show that $f_{xy}(0,0) \neq f_{yz}(0,0), where$

$$f(x,y) = \begin{cases} x^2 \tan^{-1} \left(\frac{y}{x}\right) - y^2 \tan^{-1}\frac{x}{y} & \text{if } xy \neq 0 \\ 0 & \text{if } xy = 0 \end{cases}$$

Q30. Find the shortest distance of any point (x, y, z)on the plane $lx + my + nz = p$ from the origin (0,0,0).

PAPER –II (DIFFERENTIAL EQUATION)

1. Solve $(D^4 - 2D^3$+5D^2-8D+4)y=0
2. Solve $(D^2 + 1)^3(D^2 + D + 1)^2y = 0$
3. Solve $(D^2 - 4D + 4)y = x^2e^{2x}sin2x$
4. Solve $\frac{d^4y}{dx^4}$+$m^4y = 0$,m>0
5. Solve $D^2y - 3Dy + 2y = coshx$
6. Solve $(D^3 + 1)y = 3 + e^{-x}$
7. Solve $(D^3 + 1)y = \sin(2x - 1)$
8. Solve $(D^2 - 4D + 4)y = x^2 + sin2x$
9. Solve $(D^3 + 3D^2 + 2D)y = xe^x$
10. Solve $(D^4 + 2D^2 + 1)y = x^2cosx$
11. Solve $x^3\frac{d^3y}{dx^3}$+6$x^2\frac{d^2y}{dx^2}$+4x$\frac{dy}{dx}$-4y=$(logx)^2$
12. Solve ($\frac{dy}{dx}$)$^2(x^2 - a^2) - 2\left(\frac{dy}{dx}\right)xy + y^2 - b^2 = 0$
13. Solve i) $x^3\frac{d^3y}{dx^3}$+3$x^2\frac{d^2y}{dx^2}$+x$\frac{dy}{dx}$+8y=65sin(logx)

    ii) $x^3\frac{d^3y}{dx^3}$+3$x^2\frac{d^2y}{dx^2}$+x$\frac{dy}{dx}$+8y=65cos (logx)
14. Solve $(x^2D^2 - xD + 1)y = 2logx$
15. Solve $(x + 1)^2\frac{d^2y}{dx^2}$+(x+1) $\frac{dy}{dx}$=(2x+3)(2x+4)
16. Solve (1+$e^{\frac{x}{y}}$)$dx + e^{\frac{x}{y}}$(1- $\frac{x}{y}$)dy=0
17. Solve $(x^2 + y^2 - a^2)xdx + (x^2 + y^2 - b^2)ydy = 0$
18. Solve y(xy+2x^2y^2)$dx + x(xy - x^2y^2)dy = 0$
19. Solve (1+xy)ydx+(1-xy)xdy=0
20. Solve y=2px-p^2x
21. Solve p=tan(x- $\frac{p}{1+p^2}$)
22. Solve xp^2=$(x-a)^2$

23. Solve the Clairaut's Equation $(y')^2-xy'+y=0$
24. Solve $(p-1)e^{4x}+p^2e^{2y}=0$
25. Find the General and Singular solution of

i) $x^3p^2+x^2yp+a^3=0$

ii) $3p^2e^y-px+1=0$

26. Find the Complete and Singular solution of differential equation $p^2(x^2-a^2)-2pxy+(y^2-b^2)$
27. Solve the Differential Equation
$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos[\log(1+x)]$
28. Reduce the Equation
$2x^2y\frac{d^2y}{dx^2}+4y^2=x^2(\frac{dy}{dx})^2+2xy\frac{dy}{dx}$ to Cauchy form
by substituting $y=z^2$ and solve it.
29. Solve the Differential Equation
$xy^3(ydx+2xdy)+(3ydx+5xdy)=0$
30. Prove If $\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}$ is a function of y alone,say g(y)
then $e^{\int g(y)dy}$ is an integrating factor of Mdx+Ndy=0

Paper III- Statics

1. If the greatest possible resultant of two forces $\vec{P}$ and $\vec{Q}$ acting at a point is n times the least, show that the angle between them when their resultant is half of their sum is $\cos^{-1}\left(-\frac{n^2+2}{2(n^2-1)}\right)$.
2. Let two forces of magnitudes P & Q act at such an angle that magnitude of their resultant is P. Show that if P is doubled, the new resultant is at right angles to the force of magnitude Q and its magnitude will be $\sqrt{4P^2-Q^2}$.
3. Two forces of magnitude P+Q and P-Q make an angle 2α with one another, and their resultant makes an angle θ with the bisector of the angle between them. Show that P $\tan\theta$ = Q tan α.

4. The resultant of two forces $\vec{P}$ and $\vec{Q}$ acting at an certain angle is of magnitude X and that of $\vec{P}$ and $\vec{R}$ acting at the same angle is also of magnitude X. The resultant of $\vec{Q}$ and $\vec{R}$ again acting at the same angle is of magnitude Y. Prove that P = $\sqrt{X^2+QR} = \frac{QR(Q+R)}{Q^2+R^2-Y^2}$. Prove also that if P + Q + R= 0, then X=Y.

5. The magnitude of resultant of two forces actiong at angle θ is R. When they act at angle $\frac{\pi}{2}-\theta$, their resultant is of magnitude $\frac{R}{2}$ and when they act at angle $\frac{\pi}{2}+\theta$, their resultant is of magnitude $\frac{R}{3}$ .prove that $\theta=\tan^{-1}\frac{5}{59}$.

6. Two forces $\vec{P}$ and $\vec{Q}$ acting at a point are inclined to each other at an angle α. If $\vec{P}$ and $\vec{Q}$ are interchanged in position, the resultant turns through an angle β. Show that $\tan\frac{\beta}{2}=\frac{P-Q}{P+Q}\tan\frac{\alpha}{2}$

7. ABC is a triangle and O is its circumcentre. If AO meets BC in D, Show that the components of force AD along AB & AC are $\frac{AB\sin 2B}{\sin 2B+\sin 2C}$ and $\frac{AC\sin 2C}{\sin 2B+\sin 2C}$ respectively.

8. The resolved part of a force of magnitude 100 kg wt in a direction is half of it. Find its inclination with the force & also find the other resolved part.

9. Two forces $\vec{P}$ and $\vec{Q}$ acting at a point having resultant $\vec{R}$. If the resolved part of $\vec{R}$ in the direction of $\vec{Q}$ is of magnitude P, show that the angle between the direction of the forces is 2 $\sin^{-1}$ ($\sqrt{\frac{Q}{2P}}$). Also find the magnitude of $\vec{R}$.

10. Forces act through the angular points of a triangle perpendicular to the opposite sides whose magnitude are proportional to the cosines of opposite angles. Show that magnitude of their resultant $\vec{R}$ is proportional to $\sqrt{1-8\cos A\cos B\cos C}$.

11. ABCD is a square and O is a point dividing BC in the ratio 3:1. Find the resultant of forces of magnitudes 24,10, 18N along AB,AO,AD resp.

12. ABCDEF is a regular hexagon and forces represented in magnitude and direction by the lines AB, AC AD, AE & AF act at a point A. Find the magnitude and direction of their resultant.

13. A light string of length l is fastened to two points A & B at the same level at distance a apart. A ring of weight W can slide on the string and horizontal force $\vec{P}$ is applied to it such that it is in equilibrium vertically below B. Show that P= $\frac{aW}{l}$ and tension in the string is $\frac{(l^2+a^2)W}{2l^2}$.

14. A smooth ring of weight W is threaded on a string of length l whose ends are fastened to the points A,B on the same level and at a distance a (<l) apart. The ring is held in equilibrium by a horizontal force of magnitude W. Determine the tension in the string and the inclination of its two portions to the horizontal.

15. A body is sustained on a smooth inclined plane by two forces each equal to half the weight in magnitude, one acting horizontally and the other acting along the plane. Find the inclination of the plane to the horizontal.

16. A weight W is supported on a smooth plane of inclination α to the horizontal by a force whose line of action makes an angle 2α with the horizontal. If the pressure on the plane be arithmetic mean of the weight and the force, show that $\alpha = \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$.

17. If AD is the altitude of triangle ABC, show that the force $\vec{AD}$ acting along AD has components $\frac{a^2+b^2-c^2}{2a^2}$AB and $\frac{c^2+a^2-b^2}{2a^2}$AC along AB and AC resp.

18. A body of mass 26 kg is suspended by two strings 5 cm and 12 cm long, their other ends being fastened to the extremities of a rod 13 cm long. If the rod be so held that the body hangs immediately below the middle point of the rod, find the tension in the strings.

19. A ring of weight W which can slide freely on a smooth vertical circle, is supported by a string attached to the highest point. If the thread subtends an angle θ at the centre, find the tension in the thread and the reaction of the circle on the ring.

20. A heavy uniform sphere rests on two smooth inclined planes whose inclination to the horizontal are α & β. If the pressure on the plane of inclination α is half than the weight of the sphere, prove that $\tan\beta = \frac{\sin\alpha}{2-\cos\alpha}$.

21. Two weights P and Q are suspended from a fixed point O by strings OA and OB which are kept apart by a light rod AB. If the strings make angle α & β with the rod, show that the angle θ which the rod makes with the vertical is given by

$$\tan\theta = \frac{P+Q}{P\cot\alpha - Q\cot\beta}.$$

22. Two particles of weight P & Q respectively are connected by a string which lies on a smooth circle fixed in a vertical plane. Show that if $\frac{\pi}{2}$ be the angle subtended at the centre by the string, the inclination of the chord joining the particles with horizontal is $\tan^{-1}\left(\frac{P-Q}{P+Q}\right)$.

23. A uniform plank APBQ, 12m long rests upon two supports at P & Q. The plank is just on the point of tilting up when a man of twice its weight stands at B; also when he stands at the middle point of AP. Find the positions of P & Q. Also determine the point where the man should stand in order that the pressure on two supports be equal.

24. Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ act along the sides BC,CA &AB respectively of triangle ABC. If the resultant passes through the orthocentre of the triangle, show that P sec A + Q sec B+ R sec C=0.

25. Like parallel forces 1,9,-10,8,16 and -14 kg wt act at the vertices of a regular hexagon taken in order. Find the centre of these forces.

26. Three like parallel forces of magnitude 2P+Q, 4P-Q and 8N act at the vertices of a triangle. Find P &Q if their resultant passes through the centroid of the triangle.

27. ABCD is a rectangle with AB & BC of a & b units resp. Forces of magnitude P,P act along AB & CD and forces of Q,Q act along AD & CB where P > Q. Prove that the perpendicular distance between the resultant of the forces P, Q at A and the resultant of the forces P, Q at C $\frac{Pb-Qa}{\sqrt{P^2+Q^2}}$.

28. Six coplanar forces act on a rigid body along the sides AB,BC,CD,DE,EF & FA of a regular hexagon of side 1 unit of magnitude. Their magnitudes are 10,20,30,40,P and Q units resp. Find P & Q so that the system reduces to a couple. Also find moment of the couple.

29. A system of forces acting in the plane of a triangle ABC is equivalent to a single force at A along the internal bisector of $\angle BAC$ and a couple of moment G_1. If the moments of the system about B and C are G_2 and G_3 resp. Prove that

$b\,G_2 + c\,G_3 = (b+c)\,G_1.$

30. State and prove Varigon's theorem, Parallelogram law of forces and Lami's theorem.

31. Two smooth spheres of weights W_1 & W_2 rest upon two smooth inclined planes and against each other. If α & β are the inclinations of the planes to the horizontal and θ that of line joining the centres of the spheres , then prove that

$$\tan\theta = \frac{W_1 \cot\beta - W_2 \cot\alpha}{W_1 + W_2}.$$

BCA (5TH Sem)

DISCRETE MATHEMATICS

1. For the recurrence relation S(n)-6 S(n-1) + 8 S(n-2)=0 for n≥2 & S(0)=10, S(1)=25

(i)Find generating function.

(ii)Find the sequence which satisfies it.

2. If solution of recurrence relation: a S(n) +b S(n-1) + c S(n-2) =6 is 3^n+4^n+2, find a,b,c.

3. If S(n)-6S(n-1)=5S(n-2)=0 with S(0)=S(1)=2, find generating function.

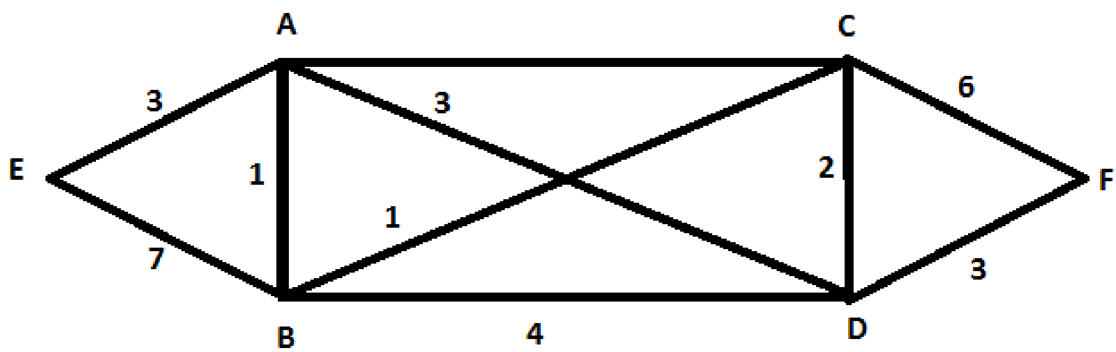
a. Using definition of generating function.
b. Using operation on sequences & their generating functions.
c. Write solution of recurrence relation.

4. Give an example of graph that has

(i)Euler Circuit but not Hamiltonian Circuit.

(ii)Hamiltonian Circuit but not Euler Circuit.

5. Find shortest path from E to F for the following graph:



6. Let G=(V,E) be a simple, connected Planar graph with more than one edge, then the following inequalities holds.

(i)$2|E|\geq 3|R|$

(ii)$|E|\leq 3|V|-6$

There is a vertex V of G such that $\deg(V)\leq 5$

7. Draw the multigraph G whose adjacency matrix A follows:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

8. Prove by induction that 21 divides 4^n+1 + 5^2n-1

9. Test the validity of the argument : If 6 is even, then 2 does not divide 7 either 5 is not prime or 2 divide 7. But 5 is not prime. Therefore, 6 is odd. "

10. Show that $f(x)=x^2+2x+1$ is $\mathbf{O}(x^2)$.

11. Show that 2^r and r! are not of same order.

12. Show that $7x^2-9x+4= O(x^2)$

13.Find the order of sum of first n positive integers.

14. By mathematical induction, prove that $1^2+2^2+3^2+ \ldots.+n^2 = \frac{n(n+1)(2n+1)}{6}\ \forall n.$

15. Prove that : $\sim(p\wedge q) \rightarrow \sim p\vee\sim q$.

16. $\sim(p\vee q) \rightarrow \sim p\wedge\sim q$

17. Check the validity of argument: If I work, I cannot study. Either I work or I passed mathematics. Therefore, I studies.

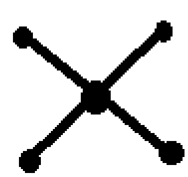
18. If n is a positive integer, then prove by mathematical induction that $\frac{n^n}{n!}<3$, when $n>3$.

19. If P(n) denotes the proposition $2^n \geq 3n$. Show that if P(n) is true, then P(n+1) is true.

20.Let P(n) =n(n+1)(n+2) divisible by 6. Is the statement P(5) true?

21.State the converse and contrapositive of the implication "If It snows tonight, then I will stay at home."

22.Does the graph shown below has Hamiltonian circuit?Justify your answer.



23. Write generating function of the sequence $s(n)= 3.4^n+2.(-1)^n+7$.

24 Construct truth table of $\sim(p\wedge q) \rightarrow \sim p\vee\sim q$.

M.Sc I

Real analysis I

1. Is set of prime numbers is countable? Justify your answer.
2. Find open balls in discrete metric space.
3. Prove that finite set has no limit point.
4. Find $\int_0^1 [5x]dx^2$.
5. Show that $\int_a^b f(x)dc = 0$ where f(x) is bounded and c is constant.

6. Prove that set of irrational numbers is uncountable.
7. Prove that set of algebraic numbers is countable.
8. Prove that finite intersection of open sets is open. Is it true for arbitrary intersection? Justify
your answer.
9. Let Y be subspace of metric space (X, d). Then F⊆ Y is closed in Y iff F=Y∩H for some closed
subset H of X.
10. Prove that closed subsets of compact sets are compact.
11. Prove that every K- cell is compact.
12. Suppose f is a bounded real function on $[a, b]$ and $f^2 \in$ R on$[a, b]$. Can we say f ∈ R? Does the
answer change if we assume $f^3 \in$ R?
13. Find $\int_0^5 (x^2 + 1) d[x]$.
14. If $\gamma: [a, b] \to R^k$ is a curve such that γ' is continuous on $[a, b]$, then γ is rectifiable and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$$

15. Let $f: [a, b] \to R$ be a bounded function and let α be a monotonic increasing function on $[a, b]$. Then $f \in R(\alpha)$ iff $\forall\, \varepsilon > 0$ there exists a partition P of $[a, b]$ such that

$$\mathrm{U}(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$$

16. Let $f: [a, b] \to R$ be a bounded function having finitely many points of discontinuity in $[a, b]$
and let α be a monotonic increasing function which is continuous at all those points where f
is discontinuous. Then $f \in R(\alpha)$ on $[a, b]$.
17. Let $f \in R(\alpha)$ on $[a, b]$ and a< c < b, then $f \in R(\alpha)$ on $[a, c]$ *and* $[c, b]$, and

$$\int_a^b f d\alpha = \int_a^c f d\alpha + \int_c^b f d\alpha$$

18. Let $f: [a, b] \to R$ be a continuous function. Let $\alpha(x) = \sum_{n=1}^{\infty} c_n I(x - s_n)$, where $c_n \geq 0$ such that $\sum_{n=1}^{\infty} c_n$ converges and $\{s_n\}$ is a sequence of distinct points in (a, b) . Then

$$\int_a^b f d\alpha = \sum_{n=1}^{\infty} c_n f(s_n)$$

Algebra I

1. If G is a group such that G/Z(G) is cyclic, then G is abelian.

2. Write down all finite abelian groups.

3. Show that no p-group is simple.

4. Prove that if G is a non abelian group of order p^3 ,where p is a prime, then Z(G) = G’.

5. Prove that if G/H $\simeq$ G/K and G is cyclic, then H =K. Also show by an example that this result does not hold if G is not cyclic.
6. Show that every finite group of order n is isomorphic to a subgroup of $S_{n.}$
7. Write down all normal subgroups of S_n, where n≥3.
8. Find the order of conjugacy class of (12) (34) in S_5.
9. Prove that the number of sylow p-subgroups of G is equal to 1 modulo p and divides the order of group.
10. Let G be a group such that o(G) = p^n and G has exactly one subgroup of order p^{n-1}. Show that G is cyclic.
11. An abelian group G has a composition series iff G is finite.
12. A group G is solvable iff G^n = {e} for some non negative integer n.
13. State and prove Cauchy's theorem for finite abelian groups.
14. Prove that quotient group of a solvable group is solvable.
15. Show that no group of order 108 is simple.
16. State and prove Jordan holder theorem for finite groups.
17. Give an example of a subnormal series of some group which is not a normal series. Justify your answer.

Differential Equations

(1) Write a general linear partial differential equation of first order.

(2)Classify $u_{xx}+ax^2u_{yy}=0$, u=u(x,y) (for parabolic, elliptic, hyperbolic).

(3) State necessary and sufficient condition for the equation u(x,y,z) dx + v(x,y,z) dy +w(x,y,z) dz=0 to be integrable.

(4) Consider the initial value problem $y'=y+\lambda x^2 \sin y$, y (0)=1, where is some real parameter, $|\lambda| \leq 1$. Show that the solution of this problem exists for $|x| \leq 1$.

(5) Show that the condition that the surfaces F9x,y,z)=0, G(x,y,z)=0 touch is that $F_x:G_x=F_y:G_y=F_z:G_z$.

(6) Solve: $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2\ (x-y)} = \frac{dz}{z(x^2+y^2)}$

(7) Solve: r+s-2t=p+2q.

(8) Solve: $(D^3-3DD'+D+1)z=e^{2x+3y}$

(9) State and prove local existence theorem.

(10) Show that $f(x,y)=\sqrt{y}$ does not satisfy a lipshitz condition on R:$|x| \leq 1$, $0 \leq y \leq 1$, whereas f satisfies a lipshitz condition on any rectangle R: :$|x| \leq a$, $b \leq y \leq c$, where a,b,c> 0.

(11) Consider the differential equation $\frac{dy}{dx}$ =2x(y+1) along with initial condition y(0)=0.

(i)Determine nth successive approximation $\varphi_n(x)$ for arbitrary value of n.

(ii) Show that $\lim_{n\to\infty} \varphi_n(x)$ exists & is the solution of the given initial value problem.

(12) Show that every initial value problem for the equation $\frac{dy}{dx} = f(x)$ p(cos y) + g(x) q(sin y) has a solution which exists for all real values x, where f,g are continuous functions for all real x & p,q are polynomials.

(13) Show that the direction cosines of the tangent at point (x,y,z) to the conic $ax^2+by^2+cz^2=1$, x=y=z=1 are proportional to by-cz, cz-ax, ax-by.

(14) Find the orthogonal trajectories on the sphere of its intersection with the paraboloids being a parameter.

(15) A Pfaffian differential equation $\overrightarrow{X}.d\vec{r} = 0$ in three variables in integrable iff $\overrightarrow{X}.$ curl$\vec{X}$ =0.

(16) Verify the given equation is integrable or not & hence find its primitive :

yz (y+z) dx +xz(x+z) dy + xy(x+y) dz=0.

(17) Solve the differential equation: y(x+4) (y+z) dx – x(y+3z) dy + 2xy dz= 0.

(18) Verify the equation: (y^2-z^2) dx + (x^2-z^2) dy +(x+y)(x+y+2z) dz = 0 is integrable and find its integral by Natani's Method.

(19) Find general equation of the differential equation: p cos(x+y) + q sin(x+y)=z.

(20)Reduce the equation :

$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}$ to canonical form, and hence solve it.

(21) $(D^2-D')(D-2D')z = 3e^{2x+y}+xy$

Number theory

1 Find the l.c.m of 858 & 325.

2 Find the number of positive integers less than or equal to 3600 which are relatively prime to 3600.

3 Prove that the number $\sqrt{2}$ is irrational .

4 Find the value of n≥1 for which 1!+2!+3!+…n! is perfect square.

$\left[\frac{[x]}{m}\right]=\left[\frac{x}{m}\right]$ if m ε **N** .

5 Given integers a & b not both of which are zero , prove that there exist integers x & y such that : gcd (a,b)=ax+by

6 State &prove Chinese remainder theorem

7 State and prove fundamental theorem of arithmetic.

8 Use Euclidean algorithm to find integers x,y,z such that (56,72)=56x+72y.

9 Find the general solution of the Diophantine equation 3x+7y=5.

10 If p is prime and a is any integer than $a^p \equiv a(mod p)$.) If p is an odd prime then

(p-2)!≡1(modp) & 2[(p-3)!]≡-1(modp).

11 State & prove Mobius Inversion Formula .

12 Euler's function ϕ is a multiplicative function.

13 If every prime that divides n also divides m then prove that ϕ(mn)= nϕ(m). Hence show that $\phi(n^2) = n\phi(n)$ for every positive integer n.

14For any positive integer n≥1, then n= $\sum_{d/n} \phi(\frac{n}{d})$

15 Find last two digits of $3^{100.}$

16 State and prove Fermat's Christmas theorem.

17 σ(n) is odd integer iff n is either a perfect square or twice a perfect square

M.Sc II

(FIELD THEORY)

1. What are all the algebraic extensions of **C** , the field of complex numbers ?

2. What is the fixed field of $G(Q(\sqrt[3]{2})/Q)$?

3.What is the degree of the field extension O($\sqrt{15}$,$\sqrt{21}$,35) over Q ?

4. If F/E/K be field extensions such that K | F is normal extension. Is E/F a normal extension? Justify your answer.

5. Find a primitive element of Q($\sqrt{2}, \sqrt[3]{5}$) over Q.

6. Find the degree of the splitting field of $x^4 + 1$ over $\mathbf{Z_2}$.

7. Find the splitting field K of x^P-1 over Q, p being a prime number. Also find [K:Q].

8. Prove that any two algebraic closures of field F are F- isomorphic.

9. Let f(x) and g(x) be irreducible polynomials over a field F such that g.c.d (deg f(x), deg g(x))=1. If 'a' is a root of f(x) in some extension of F, prove that g(x) is irreducible over F(a).

10. State and prove Kronecker's theorem.

11. A polynomial of degree $n \geq 1$ over a field F cannot have more than n roots in any field extension of F.

12. Give an example to show that an algebraic extension need not be finite.

13. An irreducible polynomial f(x) over a field F of characteristic p>0 is inseparable f(x)=F[x^P].

14. If a Ɛ k is algebraic over F then prove that F(a)=F[a].

15.Find the splitting field of $x^5-3x^3+x^2-3$ over **Q**. Also find its degree & a basis over **Q.**

16. A field is finite iff its multiplicative group is cyclic.

17. Find a field with 32 elements.

18. State and prove Lagrange's theorem on primitive element. Also, find a primitive element Of Q($\sqrt{2}, \sqrt[3]{5}$) over Q.

19. Find fixed field of C_H where H= G(C/R).

20. If H is a finite subgroup of Aut(K), then [K:K_H]=O(H).

21. Let K=Q($\sqrt{2}$). Prove that Q is fixed field under Aut(K).

TOPOLOGY

1 Define door space and give an example of a door space.

2 Cofinite topology on an infinite set in Hausdorff. Prove or Disprove.

3 Let O and C be open and closed subsets of X resp. Show that O-C is open in x and c-o is closed in x

4 If T and T^I are two topology on a set X s.t T ⊊ T^I what can say about the subspace topology on Y.

5 Give an example of space in which sub-space topology≠Ordered Topology.

6 Prove that the function f:R→ R^W defined as

F(t) =(t, t,t,t,_ _ _ _ _ _ _ _ _ ,∞ _ _ _ times) is continuous if we assume Product topology on R^W and discontinuous if we assume box topology on R^W.

7 Let X be a Hausdorff space and Abe a subset of X. Prove that x∈ A^I Iff every neighbourhood of x contains infinitely many topological space X.

Show that Intersection of all closed sets containing A=$A^I \cup A$.

8 Let Y be a subspace of X. Show that a set A is closed in Y iff it equals The Intersection of a closed of X with Y.

9 What is Relation between Uniform, Product and Box Topology.

10 U is open in Metric Topology induced by d iff for each y∈U ∃ □ > 0 s.t □$_□$ (y,□) ⊂U.

11 State and Prove Uniform limit theorem .

12 Let X be □$_2$ iff diagonal D = {□ × □/□ ∈ □} is closed in X× □.

13 Let X and Y be topological spaces. Show that f: X→ □ is continuous iff

Iff f(Ā)⊂$\overline{□(□)}$ ∀ A ⊂ X

14 Let A be connected subset of a topological space X, then Show that Ā is connected.

15 Prove that Union of a collection of connected subspaces of X that

Have a point in common is connected.

16 Rational numbers are connectd. Prove or Disprove.

17 Finite Cartesian Product of connected space is connected.

18 Prove that a compact subspace of a hausdorff space is closed.

19 A subspace A of □$^{□}$ is compact iff it is closed and bounded under

Euclidean metric d and square metric □.

20 Prove that Cantor set is compact.

21 state and prove urysohns lemma

22 show that every completely regular space is regular

23 state and proveTietz extension theorem

24 Product of finitely many compact spaces is compact.

Probability and Mathematical Statistics I

1. Give merits and demerits of mode.
2. Discuss Box and Whisker plot.
3. Define lines of regression. Why are there two lines of regression?
4. Explain the concept o conditional probability in brief. Also state law of total probability and Baye's theorem

5. A random variable X takes values 0, 1, 2, 3, 4 with equal probabilities. Find mean and variance of X.

6. Explain various measurement scales.

7. Explain the advantages of graphical representation of a frequency distribution. Develop a frequency distribution with the help of a data set of your choice and draw :

(i) a histogram (ii) a frequency polygon (iii) cumulative frequency diagrams

8. Given below is the cumulative frequency distribution of 140 candidates obtaining marks in a certain examination.

Marks	10	20	30	40	50	60	70	80	90	100
C.F.	140	133	118	100	75	45	25	9	2	0

Calculate mean, median and mode

9. The first four moments of distribution about the value 4 of the variable are -1.5, 17, -30 and 108. Find first four moments about mean, $\square_1$ □□□ $\square_2$.

10. Show that $\square_2 \geq 1$ where $\square_2$ is measure of kurtosis. Also discuss the case when equality holds.

11. Define Dispersion. What are requisites of good measures of Dispersion? In the light of these, compare various measures of dispersion.

12. Show that correlation coefficient is independent of change in origin and scale.

13. Find correlation coefficient from the following results.

$$n=10, \sum \square = 140, \sum \square = 150, \sum(\square - 10)^2 = 180, \sum(\square - 15)^2 = 215$$
$$\sum(\square - 10)(\square - 15) = 60$$

14. Calculate rank correlation coefficient for the following data.

X	64	68	50	75	80	75	55	40	64	64
Y	58	62	45	68	60	68	50	48	70	81

15. Find angle between two lines of regressions. Discuss under what situation two lines of regression coincide

16. Discuss two measures of association in a 2×2 contingency table.

17. Three numbers are chosen at random from 1, 2, 3, ..., n. What is the probability that they will form an arithmetic progression?

18. State and prove Boole's inequalities.

19. From a city Population, the probability of selecting a male or smoker is $\frac{7}{10}$, a male smoker is $\frac{2}{5}$ and a male if smoker is already selected is $\frac{2}{3}$. Find the probability of selecting (i) a non-smoker (ii) a male (iii) a smoker, if a male is first selected.

20. Explain what is meant by a random variable? Distinguish between a discrete and a continuous random variable.

21 Let X be a positive integer-valued random variable. Then Show that

(i) $\square(\square) = \sum_{\square=1}^{\infty} \square(\square \geq \square)$

(ii) $\square(\square) = \int_0^{\infty} (1 - \square(\square))\square\square$, where F(x) is c.d.f. of X

22. Define moment generating function of a random variable. For a random variable X taking the value the value n with probability $\frac{1}{2^{\square}}$, n=1, 2, 3,......... Find the moment generating function and hence find mean and variance of X.

23. State and prove Lindeberg- Levy Central Limit Theorem

24. Two fair dice are thrown. If X is the sum of the numbers shown up, then by using Chebyshev's Inequality, show that $\square(|\square - 7| \geq 3) \leq \frac{35}{54}$. Also calculate the exact value of $\square(|\square - 7| \geq 3)$

25. Given that (AB)=30, $(\square\square)$=5, $(\square\square)$ =10, $(\square\square)$ =5, Calculate coefficient of colligation. Hence find coefficient of association.

Linear programming

(1)What do you understand by shadow prices?

(2) How are variables that are ”unrestricted in sign”deal with while solving a LPP?

(3) Define Paradox in a Transportation Problem and state a sufficient condition for paradox to occur in the problem $\square_1 \; \square_2 \; \square_3$

(4) Express this assignment problem as LPP $\begin{matrix} \square_1 \\ \square_2 \\ \square_3 \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 1 & 4 \end{bmatrix}$

(5) Construct a graphical / numerical example to show that there can be two disjoint sets A and B such that A and AUB are convex sets, but B is not convex set.

(6) Let S= $\{(\square_1, \square_2): \square_1 - \square_2 \leq 4,\ \square_1 + \square_2 \geq -3, \square_2 \leq 8\}$. Find all the extreme points of S and represent (2,1) as convex combination of the extreme points.

(7) Check whether the given functions are convex or not.

(i) X= $\{(x,y) / y-3 \geq -\square^2, x \geq 0, y \geq 0\}$

(ii) S = $\{(x,y) / 3\square^2 + 2\square^2 \leq 6\}$

(8) Prove that if set of feasible solution of a LPP is non empty, then it has at least one basic feasible solution .

(9) In a simplex iteration i.e.moving from one bfs $\square_{\square}$ to another bfs $\widehat{\square_{\square}}$, a vector that leave the basis can not re-enter in the very next iteration.

(10) find the inverse of the matrix using simplex method $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

(11) State and prove Basic duality theorem.

(12) Find an optimal feasible solution of Min. Z = $\square_1 - \square_2$

s.t. $2\square_1 + \square_2 \geq 2$

$\square_1 + \square_2 \leq -1$

$\square_1 \geq 0, \square_2 \geq 0$ by solving its dual problem .

(13) Solve by revised simplex method

Max. Z = $4\square_1 + 3\square_2$

s.t. $\square_1 + \square_2 \leq 8$

$2\square_1 + \square_2 \leq 10$

$\square_1 \geq 0, \square_2 \geq 0$

(14)Prove that the number of basic variables of the general transportation problem at any stage of feasible solution must be m+n-1.

(15) Every minor of coefficient matrix in standard transportation problem is unimodular.

(16) Solve the following transportation problem by finding initial bfs using column minima method

	$\square_1$	$\square_2$	$\square_3$	$\square_4$	Maximum availability
$\square_1$	2	1	3	4	30
$\square_2$	5	6	1	2	40
$\square_3$	3	1	-2	2	60
Minimum requirement	20	20	30	30	

(17) Does Hungarian method to solve Assignment problem terminate after a finite no. of iterations? Justify your answer.

(18) Five persons have to be assigned to five machines. The assignment costs are given below in the table as follows: For some technical reasons $\square_1$ can not operate machine $\square_3$ and $\square_3$ can not operate machine $\square_4$. find the optimal assignment.

	$\square_1$	$\square_2$	$\square_3$	$\square_4$	$\square_5$
$\square_1$	5	5	4	2	6
$\square_2$	7	4	2	3	4
$\square_3$	9	3	5	4	3
$\square_4$	7	2	6	7	2
$\square_5$	6	5	7	9	1